Internal Force Analysis of a Planar Truss Based on Electrical Measurement Experiments and Theoretical Calculations

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Abstract: *In this paper we aim to study the distribution law of internal forces in planar truss structures by selecting a classical planar truss as our research object and compare their theoretical calculation with electrical measurement experiment. We show there is a difference between the theory calculation value and the experimental value which was due to the joint structure of the truss, the machining accuracy of the members, the self-weight of the members and the accuracy of the measurement equipment. Designing an actual truss structure needs to reserve enough safety margin to ensure the structural safety and do regular inspection on structural deformation when using to detect problems at an early stage.*

Keywords: *planar truss; electrical measurement experiment; theoretical calculation*

1. Introduction

Truss structures have been widely used in large public buildings (exhibition hall, gymnasium or bridge), owing to its advantages of clear force transmission, high material utilization and flexible layout[1]. If we assume that the members only experience axial tension and compression; the joints are smooth hinges; and the loads only act on the joints, then we can calculate the internal forces through structural mechanics theory[2]. In reality however, because of the rigidity of the truss joint, deviation of axis for each member and most of the load being non-joint loads, there would be differences between the actual internal forces/deformations with those theoretically calculated[3] but the actual distribution of internal forces within a structure will affect the safety performance of the truss structure. Therefore understanding the difference between actual internal forces and theoretical internal forces and applying them when designing truss structures has great application values in engineering practice. We chose a classic statically determinate truss model as an example in our research to explore the internal force distribution characteristics of the truss under lab condition and do error analysis.

2. Theoretical Analysis of the Planar Truss

2.1 Theoretical Model of the Truss

A symmetric, planar, statically determinate truss consisting of 13 members is used (Figure 1), with a fixed hinge on one end and a roller hinge on another end: we simplify the external loads as concentrated forces acting at joints. The material properties of the members used in our experiment are given as follows: Elastic Modulus E = 206 GPa and Poisson's Ratio $\mu = 0.26$.

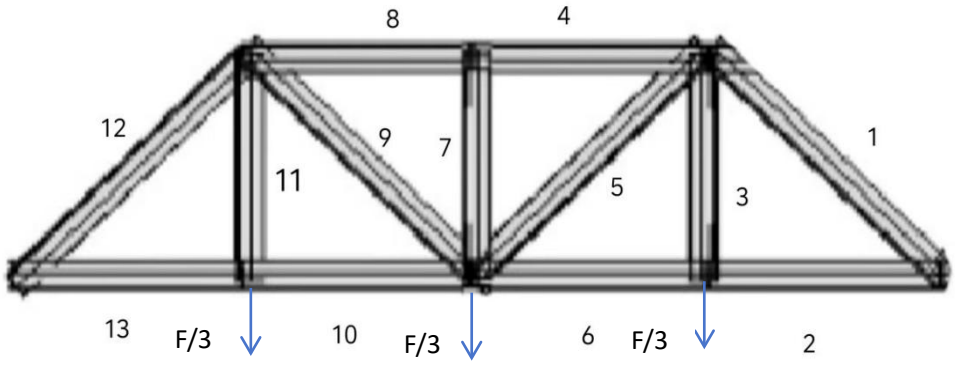


Figure 1 Truss Structure and Member Numbering

2.2 Theoretical Calculation of Internal Forces

Based on the schematic diagram of the truss structure, we compute the internal forces of every member by either the method of joints or the method of sections: Starting with the support joint, the method of joints uses the force equilibrium conditions (in horizontal direction $\sum Fx=0$ and in vertical direction $\sum Fy=0$) at a joint to solve the internal forces of the members one after another; Cutting out some parts of the truss structure, the method of sections utilizes the moment equilibrium condition ($\sum M=0$), solving the internal force of a certain member.

Take the truss shown in Figure 1 for example, when apply three concentrated loads symmetrically, we get the calculation result about the internal forces of the key members as below: The internal forces of members 1 and 12 are $\frac{1}{\sqrt{2}}$F, the internal forces of members 4 and 8 are $\frac{5\sqrt{2}}{12}$F, member 7 is a zero-force member, its internal force is 0. It can know by theoretical calculation, the distribution law of the internal forces of this truss has been clarified with which the experiment and simulation have baseline data.

3. Experimental Study on the Planar Truss by Electrical Measurement

Basic principle: The method utilizes resistance strain gauges to measure the linear strain on the surface of a member, and determine its surface stress state according to the stress-strain relationship -- an experimental stress analysis method. Resistance strain gauges are attached to the surface of the member under test; when it deforms, the resistance value of the resistance strain gauge also changes. These resistance changes are converted by a resistance strain indicator to change in voltage (or current), which will then be transformed into strains or as voltage (or current) signals proportionate with the strain. Recorder records them so that we get the measured strain or stress.

3.1 Experimental Equipment and Truss Model

A multifunctional mechanics experimental bench (specially designed truss members and connectors) was used where we are able to assemble cantilever trusses or simply supported trusses. There are three mounting support positions on the column of the right-angle rigid frame of the experimental bench; two of them are truss assembly position. Two semicircular supports slotted at $\pm 45°$ and $0°$, plum-blossom-shaped connecting discs with eight equal spacing slots and matching truss members of two lengths have been provided by this experimental bench. The semicircular supports were mounted onto the column, and both horizontal adjusting bolts on the rigid frame were adjusted and fixed such that the rigid frame does not rotate anymore – we can assemble cantilever trusses of various structures and number of joints starting from the supports. In Fig. 2 and 3 we present two examples of these assemblies. A structure chosen for our experiment is the simply supported truss presented in Figure 4.

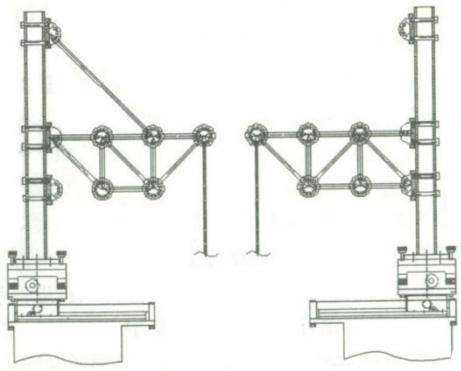


Figure 2 Schematic Diagram of the Cantilever Truss Structure

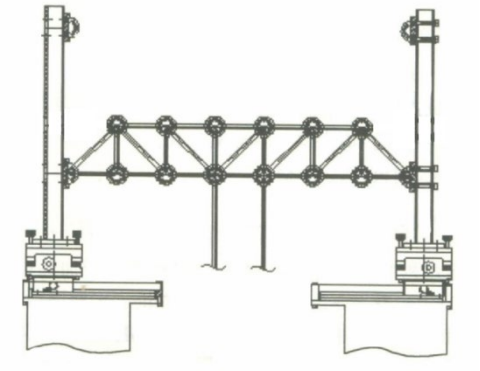


Figure 3 Schematic Diagram of the Simply Supported Truss Structure

The overall dimension of truss members is shown in Figure 5. To remove the influence caused by the initial curvature of the specimen and connector constraints, two strain gauges are located at both sides of the middle of each truss member; when measuring we take an average between the two strain gauge values for the strain value of a member.



Figure 4 Planar Truss Structure Used in the Experiment

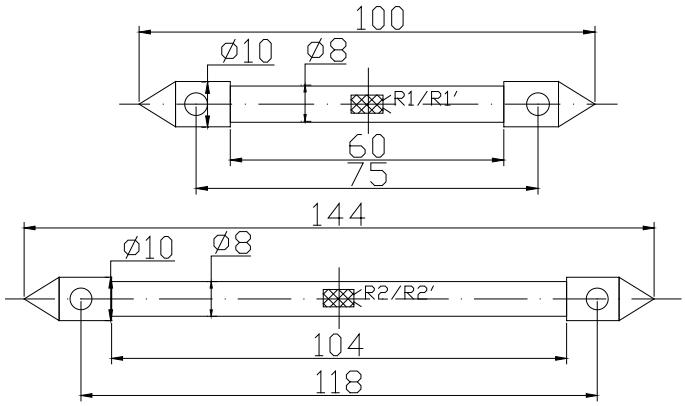


Figure 5 Schematic Diagram of the Overall Dimensions of the Truss Members

3.2 Experimental Scheme and Data Acquisition

The stepwise loading method is adopted during the experiment with a range from 100 N to 1200 N. The data are recorded after stabilizing for 30 s so as to reduce the influence of loading impact. The strain value of every member was collected based on each load level (Table 1), which shows that the axial strain values of the members increase linearly when the external load increases, conforming to the basic law (Figure 6).

Table 1 Strain Values of Each Truss Member under Stepwise Loading Conditions (Unit: $\mu\varepsilon$)

External Force	1	2	3	4	5	6	7	8	9	10	11	12	13
100N	-6	5	4	-7	2	0	0	-5	2	3	1	-6	4
200N	-12	10	40	-12	1	1	0	-9	1	7	4	-12	8
300N	-17	15	17	-17	0	1	0	-13	0	11	8	-19	12
400N	-23	21	24	-21	0	1	0	-17	-1	15	13	-25	18
500N	-28	26	31	-26	-1	2	-1	-20	-2	20	18	-32	22
600N	-32	28	38	-31	-2	3	-1	-24	-3	25	23	-38	26
700N	-35	30	44	-36	-3	3	-2	-28	-4	31	28	-44	30
800N	-39	34	51	-40	-3	2	-2	-32	-5	35	33	-51	35
900N	-43	38	56	-43	-3	1	-3	-36	-5	40	36	-57	39
1000N	-46	40	60	-51	-4	0	-4	-40	-5	44	40	-63	44
1100N	-48	39	65	-56	-5	0	-4	-45	-5	48	43	-69	50
1200N	-50	37	70	-62	-5	0	-4	-48	-5	53	47	-75	57

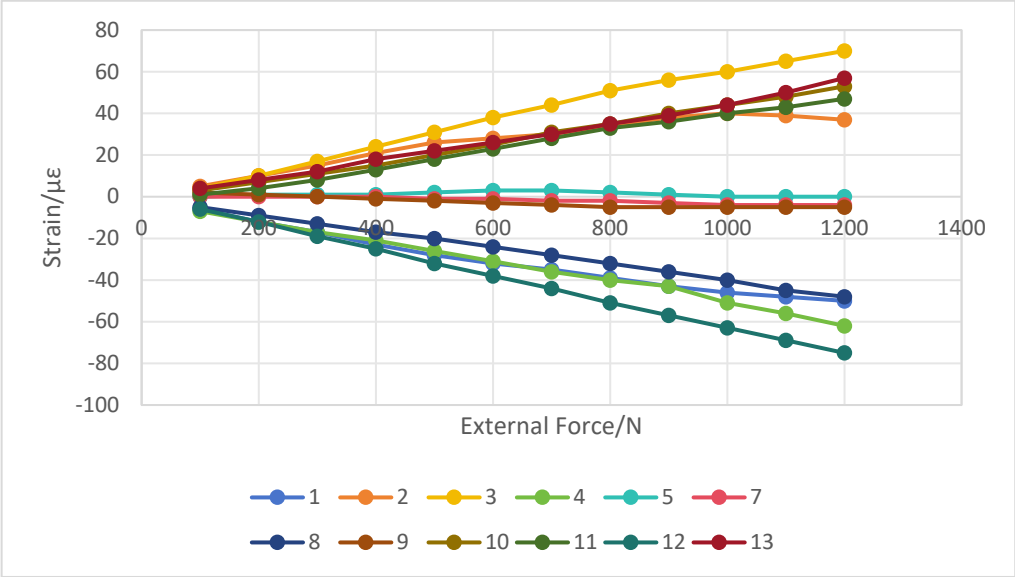


Figure 6 Strain distribution of each member in the truss

4. Comparison of Results and Error Analysis

4.1 Comparison of Internal Force Results

Based on a basic theory in engineering mechanics, we calculated theoretical stress value and experimental stress value on the cross section of the member based on the stress calculation formula $\sigma = \frac{N}{A}$ for axially loaded members and Hooke's law $\sigma = E\varepsilon$. Data was given as follows; tables 2 and 3 and error calculations are presented (Table 4)

Table 2 Theoretical Stress Values of Each Truss Member under Stepwise Loading Conditions (Unit: MPa)

External Force Member	1	2	3	4	5	6	7
100N	-1.407	0.995	0.657	-1.172	0.469	0.995	0
200N	-2.813	1.989	1.313	-2.344	0.938	1.989	0
300N	-4.220	2.984	1.970	-3.515	1.407	2.984	0
400N	-5.626	3.979	2.626	-4.687	1.876	3.979	0
500N	-7.033	4.974	3.283	-5.859	2.345	4.974	0
600N	-8.439	5.968	3.939	-7.031	2.813	5.968	0
700N	-9.846	6.963	4.596	-8.202	3.282	6.963	0
800N	-11.252	7.958	5.252	-9.374	3.751	7.958	0
900N	-12.659	8.952	5.909	-10.546	4.220	8.952	0
1000N	-14.065	9.947	6.565	-11.718	4.689	9.947	0
1100N	-15.472	10.942	7.222	-12.890	5.158	10.942	0
1200N	-16.878	11.937	7.878	-14.061	5.627	11.937	0

Table 3 Experimental Stress Values of Each Truss Member under Stepwise Loading Conditions (Unit: MPa)

External Force Member	1	2	3	4	5	6	7
100N	-1.236	1.03	0.824	-1.442	0.412	0	0
200N	-2.472	2.06	2.06	-2.472	0.206	0.206	0
300N	-3.502	3.09	3.502	-3.502	0	0.206	0
400N	-4.738	4.326	4.944	-4.326	0	0.206	0
500N	-5.768	5.356	6.386	-5.356	-0.206	0.412	-0.206
600N	-6.592	5.768	7.828	-6.386	-0.412	0.618	-0.206
700N	-7.21	6.18	9.064	-7.416	-0.618	0.618	-0.412
800N	-8.034	7.004	10.506	-8.24	-0.618	0.412	-0.412
900N	-8.858	7.828	11.536	-8.858	-0.618	0.206	-0.618
1000N	-9.476	8.24	12.36	-10.506	-0.824	0	-0.824
1100N	-9.888	8.034	13.39	-11.536	-1.03	0	-0.824
1200N	-10.3	7.622	14.42	-12.772	-1.03	0	-0.824

Table 4 Errors between Theoretical and Experimental Stress Values of the Members

Error	1	2	3	4	5	6	7
100N	12.12%	3.55%	25.51%	23.06%	12.14%	100.00%	-
200N	12.12%	3.55%	56.89%	5.48%	78.03%	89.65%	-
300N	17.01%	3.55%	77.81%	0.38%	100.00%	93.10%	-
400N	15.79%	8.72%	88.27%	7.70%	100.00%	94.82%	-
500N	17.98%	7.69%	94.54%	8.58%	108.79%	91.72%	-
600N	21.89%	3.36%	98.73%	9.17%	114.64%	89.65%	-
700N	26.77%	11.25%	97.23%	9.59%	118.83%	91.12%	-
800N	28.60%	11.98%	100.03%	12.10%	116.47%	94.82%	-
900N	30.02%	12.56%	95.24%	16.01%	114.64%	97.70%	-
1000N	32.63%	17.16%	88.27%	10.34%	117.57%	100.00%	-
1100N	36.09%	26.58%	85.41%	10.50%	119.97%	100.00%	-
1200N	38.98%	36.15%	83.04%	9.17%	118.30%	100.00%	-

4.2 Analysis of Error Sources

In the truss structure, there is a considerable difference between the measured stresses and the theoretically calculated values, which may be attributed to the following reasons:

Joint influence: Real joints have semi-rigid hinges with engagement clearances and friction that carry a portion of bending moments (not in accordance with an ideal hinge)[9];

Member deviation: The axes of the members are not absolutely straight and have initial curvature, which generates additional bending moments;

Load influence: Member self-weight and non-joint loads are present in the experiment, while the theoretical and simulation analyses ignore secondary loads;

Equipment accuracy: Measurement errors of the strain indicator and deviations in strain gauge bonding positions affect data accuracy.

5. Conclusion

The internal force distribution in an actual truss structure differs from the theoretically calculated value; we can get the actual internal force distribution of the truss through the electrical measurement experiment. There are several reasons that lead to this difference including joint structure of the truss, machining accuracy of the members, self-weight of the members, as well as the accuracy of the measurement equipment. In an actual truss structure, there exist larger stress on some members than the theoretical values while the others have less stress compared with the theoretical values. It's necessary to reserve enough safety margin when designing to guarantee structural safety and do regular inspection of structural deformation during using to identify problems earlier.

Fund Projects

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