

Design of Grey Prediction Controller Based on Adaptive Grey Wolf Optimizer

Guangsheng Li*

Wuhan College of Arts & Science, Wuhan, 430345, China

*Corresponding author: 17363316673@163.com

Abstract: Although there are theoretical advantages of grey prediction control when dealing with systems having poor information; but they have problems such as experience-dependent parameter tuning, fixed model structure and vulnerable to local optima during optimization. We propose integration of the adaptive grey wolf optimizer into the parameter collaborative tuning process of the grey prediction controller resulting in an integrated design approach. Algorithm level: analyse the discrepancy between the linear decay of the convergence factor and the search state in the standard grey wolf algorithm by suggesting an adaptive adjustment based on the variance of population distribution. Prove convergence via Markov chain theory without increasing additional computational overhead up to a constant factor magnitude. Modeling level: Construct a dynamic grey prediction module in the forward channel of the controller (using a rolling window and online parameter adjustment) tracking time-varying characteristics and designing an online correction strategy separating deviation temporal features suppressing prediction inaccuracies. Parameter optimization level: Adjust all adjustable parameters of the predictor and the controller into one optimization vector building a weighted objective function. Collaboration optimization lead us to use the Lyapunov method deriving sufficient conditions ensuring the stability of the closed-loop system. Coordination match of predictive modeling and control tuning under the same framework provide an effective way for the design of grey prediction controllers.

Keywords: Adaptive Grey Wolf Optimizer; Grey Prediction Control; Parameter Collaborative Optimization; Dynamic Modeling; Convergence Analysis

Introduction

Industrial control objects usually have nonlinearities, time variation, hard to be precisely modeled, traditional control strategy based on transfer function suffers severe performance degradation due to incomplete model information. Grey prediction theory has advantage in modeling with poor information that could extract evolution trend by small amount of history data. But they tune the prediction module's parameter and the controller's parameter separately without collaboration which make the control quality sensitive to the value of parameters. Swarm intelligence optimization algorithms provide a feasible way to automatically tune the controller parameters. The grey wolf optimizer is widely used because of its simple structure but its linearly decreasing convergence factor can't adaptively adjust the balance between exploration and exploitation according to the population search state so it easily fall into local optima when solving high dimension nonconvex problems. We combined the adaptive grey wolf optimizer with the design of grey prediction controllers considering the aspect of adaptive improvement of the algorithm, dynamic predictive modeling and collaborative parameter optimization providing methodological support for control system design under poor-information conditions.

1. Mechanism Improvement and Convergence Analysis of the Adaptive Grey Wolf Optimizer

1.1 Hierarchical Interaction Mechanism and Parameter Sensitivity of the Grey Wolf Optimizer

Grey Wolf Optimizer mimics the social hierarchy and collective hunting behavior of wolf pack. In this algorithm, α , β , δ represent the top three wolves which is the current best, second-best and third-best candidate respectively while others updates their positions according to these wolves' guidance. The hierarchical interaction possesses an interesting searching guiding character forcing all

the population converging towards areas covered by the leading wolves. Solidified social hierarchy speeds up the local convergence but reduces the depth of information exchange within the population. Meaning: ω individuals only changes their positions every iteration depending upon the weighted average position of the three leading wolves, the competitive relationship among the leading wolves as well as any corrective information provided by the second-best individual about the search direction cannot be effectively utilized. Unidirectional information transfer restricts the perception of the structure of the solution space by the population. Convergence factor a uses a linear decay strategy ranging from 2 to 0, it controls the magnitude of perturbation for updating individual positions; However, there exists no inherent correlation between its changing rate and the search progress.

Based on the result of parameter sensitivity analysis, it is found that the decay rhythm of a has obvious effect on the algorithm's performance: too fast will lead to entering into the fine exploitation stage before sufficient exploration in the solution space which easily miss the global optimum region while too slow will delay the convergence process resulting in unnecessary computational overhead. Basically ignoring the real search state of the algorithm throughout its whole iteration process for such a linear decay strategy without any information feedback between the population distribution variance (the degree of individual dispersion) and the parameter value thus shows great fluctuation when facing objective functions with different characteristics. It gives us a clue as to how we can improve the construction of the subsequent adaptive regulation mechanism^[1].

1.2 Adaptive Parameter Regulation Strategy Based on Population Distribution Variance

In order to overcome the drawback of fixed decay mode of the convergence factor in the conventional algorithm, we have put forward a state-aware regulation method with population distribution variance: use the fitness variance of current population in the decision space as the main measure to judge the search stage: if the variance is large then there are highly scattered individuals indicating that the algorithm is at the exploration-dominated state while they tend to cluster which is appropriate for shift into local exploitation. According to them, we map the convergence factor a nonlinearly and dynamically via the normalized variance ratio so that a closed loop exist between the parameter value and the real time distribution state of the population. Variance calculation is based on the statistical second-order moment of fitness values; normalization should be done to remove dimensional effect. Mapping function takes a sigmoid curve shape to make sure that the parameter will approach 1 when the population is dispersed or approach 0 when clustering, the slope of the transition zone can be preset according to the characteristics of the objective function.

Meanwhile, we propose the weights for the leading wolves' positions update equation by a variance-driven way that if the dispersion level of the population is large then decreasing the dominance weight of the alpha wolf can ensure independent exploration from individuals but on the contrary, when the aggregation level is high, the guiding effect of the leading wolves should be slightly enhanced so as to improve the convergence efficiency. This regulation strategy has the main advantage that the behavior mode of the algorithm can adjust itself according to the real needs during the search process which resolves the inherent conflict between exploration and exploitation in the standard algorithm. Computationally speaking, this adaptive mechanism requires just some extra calculation of the population variance and its derived indicators at every iteration compared with the total function evaluations cost of the algorithm, which is within an acceptable range^[2].

1.3 Convergence Proof and Computational Complexity Evaluation of the Improved Algorithm

Very important metric about the reliability of stochastic optimization algorithms which should be theoretically guaranteed is its convergence. According to the convergence criteria for stochastic search algorithms, by incorporating our improved grey wolf optimizer into an analysis framework based on Markov chains. Proved that if the adaptive parameters satisfy the boundedness condition and the monotonic non-increasing constraint, it will converge to the neighborhood of the global optimal solution with probability 1. In order to prove it, treat the sequence of population states as a homogeneous Markov chain over a finite state space. Construct a supermartingale sequence, apply Doob's convergence theorem, we got the sufficient conditions for algorithm convergence: take the optimal fitness value at each generation as a stochastic process; show that the expectation sequence is monotonically decreasing and bounded below, furthermore utilize the square integrable of the martingale difference sequence to get the conclusion of almost sure convergence. From this result, we can see that adding the adaptive regulation mechanism does not affect the inherent convergence property of the original algorithm, but enhance the constraint ability against abnormal search behaviors

through the parameter bounded design^[3].

Computational complexity evaluation: The time complexity of one iteration of the standard grey wolf optimizer is $O(N \times D)$: wherein N stands for the population size; D represents the dimension of decision variables. In the improved algorithm, we have to compute the population fitness variance and spatial distribution density in each generation. Both their complexities are at the order of $O(N)$ and $O(N \times D)$, similar as the core overhead of the standard algorithm. Comprehensive analysis shows that the time complexity of the improved algorithm is $O(N \times D \times T)$; where T refers to the maximum number of iterations. Comparing with the standard algorithm, there will be just an extra computational cost of a constant factor level added by this approach. It suggests that our proposed adaptive strategy has achieved an effective improvement on the search performance under the control of computational resources consumption which provides a feasible basis for its applications in practical engineering scenarios.

2. Grey Prediction Modeling and Error Compensation Mechanism for Controller Design

2.1 Applicability Analysis of Grey Prediction Models in Industrial Control Scenarios

The Industrial Control Systems usually face the problems of scarce sample data, varying parameters and uncertain disturbances; traditional predictive controls based on a precise mathematical model suffer from modeling or degradation in accuracy faced with such poor-information systems. The grey prediction theory has great advantages in getting the trend information of system evolution by limited history output data without depending upon the accurate transfer function/state-space description of the controlled plant which makes it specially valuable for applications in industrial scenarios where mechanism models are hard to be established. In order to address the common time-delay and nonlinearity characteristic in control systems, the GM(1,1) model and its derivatives make ahead-of-time estimation of future outputs via an exponential fit of the accumulated generating sequence provides the necessary reference trajectory information for the controller^[4].

When using a grey prediction model as controller, there are some constraints: Common grey prediction model supposes the original sequence obeys an approximately exponential law; however, when the output of the controlled plant has large fluctuation due to external disturbance or sudden change of setpoint, the prediction accuracy of this model will deteriorate obviously. When considering the control systems with fixed sampling period and high real-time requirement, it needs the building up and updating of the prediction model to finish all the calculation in a short time, which brings certain demands on the structure of the simplified model and the efficiency of parameter identification. We have concluded that the grey prediction model can not be directly transplanted into the controller, so adjusting its structure to satisfy the real-time and anti-interference requirements of the control system is exactly what we start from for the construction of the subsequent dynamic prediction module^[5].

2.2 Construction of the Dynamic Grey Prediction Module in the Controller Structure

In order to satisfy the requirement of real-time and adaptive in the control system for prediction, we incorporate a grey prediction module with dynamic update mechanism into the forward channel of the controller. It uses a fixed-length rolling data window capturing the latest system output sequence; upon arrival of a new sampling datum, drop out the oldest one such that the modeling data will always represent the current operating state of the system. How to select the window length should balance between the anti-interference capability and the sensitivity of the model to abrupt signal changes: if too small then the model is very sensitive to noise but if too large, the model will lose its ability to follow time-varying characteristics. Actually, this value can be tuned by the response time constant of the controlled plant^[6] as needed.

Prediction model construction: Set up the range of adjusting coefficients of background value construction and accumulated generating order under dynamic module such as adapting its own by changing system operating point. Background value coefficient decides the weight of information among adjacent time instant in an accumulated sequence affecting the bias of model parameter identification; accumulated generating order determines how deep the original data transform when being generated from sequence, the higher the order has better smoothing effect on random fluctuations. Adjusting this kind of parameters online can make the prediction module could automatically fit the dynamic characteristics of systems at different operations stage to keep a relatively high degree of fitting between the predicted output and the actual system response. Its output will be the input base for

the lead correction link of the controller providing forward looking reference information for control decisions.

2.3 Online Correction and Feedback Compensation Strategy for Prediction Deviations

The dynamic grey prediction module with self-adjustment ability about its model structure will have some deviation between predicted output and actual output because of existence of unmodeled dynamics and random disturbance in the system. Accumulated deviations can make the control action deviate from desired directions if not corrected. We propose an online correction strategy which extract temporal features of deviations. Analyzing the statistics of consecutive multi-step prediction errors, we get the trend and amplitude range of the deviations then generate a correction term that modifies the predicted output in real time^[7].

Our correction method's main concept is based on separating the low- and high-frequency parts of the deviation sequence: Low frequency deviation means the trend-level differences between the model and the system, which can be compensated by a moving average estimate; High frequency deviation mainly comes from measurement noise and random disturbances, whose effect is slightly reduced due to limitation and filtering. Corrected predicted value will be compared against the actual system output inside the feedback loop (closed-loop), so that the error of the prediction module stays under control. Integrated prediction-correction greatly improves suppressing non-stationary disturbances while not changing the basic framework of the grey prediction model, providing us a stable source of prediction information for the next step collaborative parameter optimization.

3. Collaborative Parameter Optimization of the Grey Prediction Controller Driven by the Adaptive Grey Wolf Optimizer

3.1 Mathematical Modeling of the Controller Parameter Optimization Problem and Objective Function Design

Before put the grey prediction controller into actual operation, we need to set up multiple adjustable parameters which exists in both the prediction module and the controller module with coupling relationship between each other. Parameters to tune in the prediction module include: background value construction coefficient and accumulated generating order, these two parameters decide the fitting characteristic of the grey model on the system output sequence; Controller part has some conventional regulating parameters like proportional gain and integral time constant, they have direct effect on the generation of control quantity. All the parameters to tune will be formed a unified decision vector, thus create a multidimensional continuous optimization space. Due to the cross-coupling among parameters, it's hard for us to manually adjust by empirical methods so as to get globally coordinated performance^[8].

It is necessary to take all aspects of the control system's dynamic response quality and steady state accuracy when designing our objective function in this study. Combine these four indicators (rise time, overshoot, settling time, and steady-state absolute error) as a fitness function using weighted sum which weights are chosen according to different control scenarios preference. Propose a constraint handling method imposing penalties on those parameter sets beyond the stability margin limits or the actuator saturation boundaries so that it can transform into an unconstrained problem suitable for the solution framework of swarm intelligence algorithms. The proposed objective function not only considers the rapid response capability and suppresses overshoots; however, we also give considerations about the convergence accuracy under long-term operation: thus providing a rational evaluation criterion for further optimization process.

3.2 Collaborative Parameter Optimization Strategy Based on the Adaptive Grey Wolf Optimizer

Once we obtain the objective function and boundary of decision variables, we use an improved adaptive grey wolf optimizer as our parameter optimization method here. All individuals of the algorithm were first evenly distributed into the decision space where every one represents a full set of controller parameters whose fitness was decided based upon the value of the objective function obtained through the closed-loop simulation model. The adaptive parameter regulation mechanism adapts the convergence factor and the weights of the leading wolves depending on the variance of the population distribution throughout the iterations such that it has enough coverage over the solution

space at first but focuses on the refinement search around those having smaller objective function values eventually^[9].

In order to avoid trapping by local extremum in later stage of optimization, we introduce an elite opposition-based learning operator at position update phase which produces the opposition solution according to the current best individual, replacing it with better fitness value when possible (to increase the chance for the population to escape from local attraction basins). The above procedure continues until the preset maximum number of iterations has been reached or the improvement magnitude of the fitness drops below a given threshold; finally, decoding the best individual in the population into the full parameter configuration of the grey prediction controller. Through this collaboration optimization, both the parameters of prediction module and controller module can be determined under one optimization framework, overcoming the inherent deficiency of insufficient parameter coordination in stepwise tuning.

3.3 Stability Criteria of the Control System and Analysis of Dynamic Performance Degradation

Once we finish the parameter optimization process, it is necessary to verify the stability of the closed-loop system based on control theory. We employed the Lyapunov's second method here; by writing down the closed-loop system acting under the grey prediction controller into its dynamic equation (with variables being state deviations), construct a positive-definite quadratic energy function serving as a candidate Lyapunov function, derive the sufficient conditions for the asymptotic stability of the closed-loop system expressed as inequalities among the optimized parameters and the system model parameters via an inequality scaling approach on the total derivative of the energy function along the system trajectory. In our derivation, the modeling error of the grey prediction module has been regarded as a bounded disturbance term such that there are some tolerance intervals about the model mismatch^[10] when we obtain the stability criteria.

Dynamic Performance Degradation Analysis: We analyze how much the guaranteed control quality according to the optimized parameters will degrade if going out of its nominal state. In particular, considering the presence of perturbation in the gain of the controlled plant and shift in the time constant, we calculate the magnitude of deviation from the overshoot and settling time of the closed-loop system vs. the nominal ones using simulations; moreover, we have also analyzed the sensitivity of each performance indicator against the degree of mismatch. From our results, one may observe that the parameter set for the controller obtained via collaborative optimization offers a relatively large stability margin and there is a gradual degradation instead of sudden deterioration of the performance indicators, indicating that the parameter configuration acquired through the optimization has good adaptability towards model uncertainties, validating the practicality of utilizing the Adaptive Grey Wolf Optimizer as a parameter tuning tool.

Conclusion

Two main problems were considered during design of grey prediction controllers - difficult parameter tuning problem and lack of adaptivity of optimization algorithms. An Adaptive Grey Wolf Optimizer was embedded in the parameter configuration process of the controller as a whole framework including algorithm improvement, predictive modeling and collaborative optimization: For algorithm, we developed an adaptive parameter regulation strategy based on the population distribution variance which alleviated the inherent contradiction between exploration and exploitation in the standard algorithm whose convergence proof and complexity analysis proved the theoretical reliability and computational feasibility of the improved algorithm; For modeling, by combining the dynamic grey prediction module together with an online deviation correction strategy, it makes the predicted output follow change in the system's operating state so that the controller has forward-looking information of controllable quality; For parameter optimization, both parameters from prediction module and controller module are included into one optimization vector for collaborative tuning through the elite opposition-based learning operator to achieve global optimization under multi-parameter coupling conditions. Stability criteria and performance degradation analysis confirmed the usable range of the optimized parameters under model mismatch. In our future works, extending this framework to multi-input multi-output systems or integrating deep reinforcement learning methods to improve the adaptability of the prediction module to strong nonlinearity will be further explored.

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